

Neutrino oscillations and shortcuts in the extra dimension

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Santa Fe Summer Workshop
INFO 05

Outline

- Large extra dimensions: particle physics & cosmology
- Neutrinos in extra dimensions
- Neutrino oscillations & LSND dilemma
- Bulk shortcuts, neutrino oscillations & LSND
- Neutrino bulk shortcuts:
particle physics, astrophysics & cosmology
- Causality: Tales of the Timeless?

Based on H. Päs, S. Pakvasa, T.J. Weiler: hep-ph/0504096

Extra dimensions

- First ideas of extra dimensions: Theodor Kaluza (1921) & Oskar Klein (1926)



- **Fundamental ingredient of string theory**: consistently formulated in a space-time with 10 or 11 dimensions
- **Traditional picture**: extra dimensions **compactified with radii $\mathcal{O}(l_P) \sim 10^{-33}$ cm**
- **Large extra dimensions allowed, if observable world constrained on a (3+1)-brane**

P. Horava, E. Witten, 1996

→ New light on old problems in particle physics & cosmology

Large extra dimensions and the hierarchy problem

Hierarchy problem: Why is $m_{EW} \ll M_P$ stable?

Large extra dimensions:

- Gravity propagates in extra dimensions, while $3+1d$ universe is confined on a submanifold (“brane”)
- **$4d$ Planck scale** ($M_{P4d} = G_N^{-1/2} \sim 10^{19}$ GeV) \gg **fundamental**
 $(4 + \delta)d$ Planck scale $M_{P\delta d}$

generalized Gauss law:

$$M_{P4d}^2 = M_{P\delta d}^{\delta+2} V_\delta$$

- V_δ : volume of δ -dimensional extra space
- $M_{P\delta d} \sim \mathcal{O}(m_{EW}) \rightarrow$ **large scale M_P completely avoided**

N. Arkhani-Hamed, S. Dimouloulos, G.R. Dvali, 1998; I. Antoniadis, N. Arkhani-Hamed, S. Dimouloulos, G.R. Dvali, 1999; L. Randall, R. Sundrum, 1999

Consequences of large extra dimensions

Gravity is increasing strongly at energies $> R^{-1}$

$R = \sqrt[\delta]{V}/2\pi$ compactification radius

$M_{P\delta d} \sim 1 \text{ TeV} \rightarrow R \simeq \text{mm-fm}$ for $\delta = 2 - 6$

Consequences:

- Kaluza-Klein States

- Analogy: particle in a box, momenta of bulk fields are quantized
- 4d perspective: **tower of Kaluza-Klein excitations:**

$$m_n^2 = \frac{n^2}{R^2}$$

- Dark matter candidates

- TeV **black hole production at the LHC!**

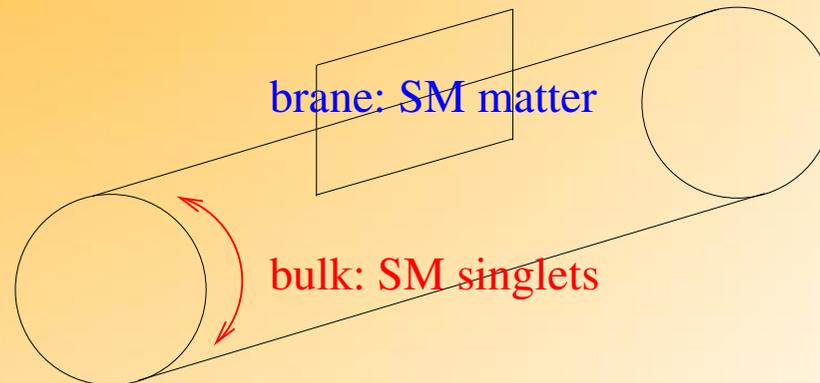
- Alternative to compactification: warping of extra dimension
[L. Randall & R. Sundrum, 1999](#)

Large extra dimensions and neutrino masses

- **No large scale** → **no seesaw** suppression of neutrino masses
- However: string theories → singlet fermions in the bulk (e.g. superpartners of moduli fields) → ν_R

→ **small Dirac neutrino masses** from **volume-suppressed couplings** to ν_R in the bulk:

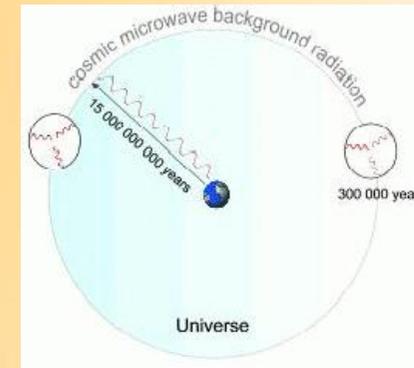
$$m^D = \frac{vY}{\sqrt{2V_\delta} M_{P\delta d}^\delta} = v \frac{Y}{\sqrt{2}} \frac{M_{P\delta d}}{M_{P4d}} \quad \text{suppression factor: } M_{P\delta d}/M_{P4d}$$



N. Arkhani-Hamed, S. Dimouloulos, G.R. Dvali, J. March-Russel, 1998; K.R. Dienes, E. Dudas, T. Gherghetta, 1999; Y. Grossman, M. Neubert, 2000; S.J. Huber, Q. Shafi, 2002; G. Bhattacharyya, H.V. Klapdor-Kleingrothaus, H. Päs, A. Pilaftsis, 2002

Bulk shortcuts and the horizon problem

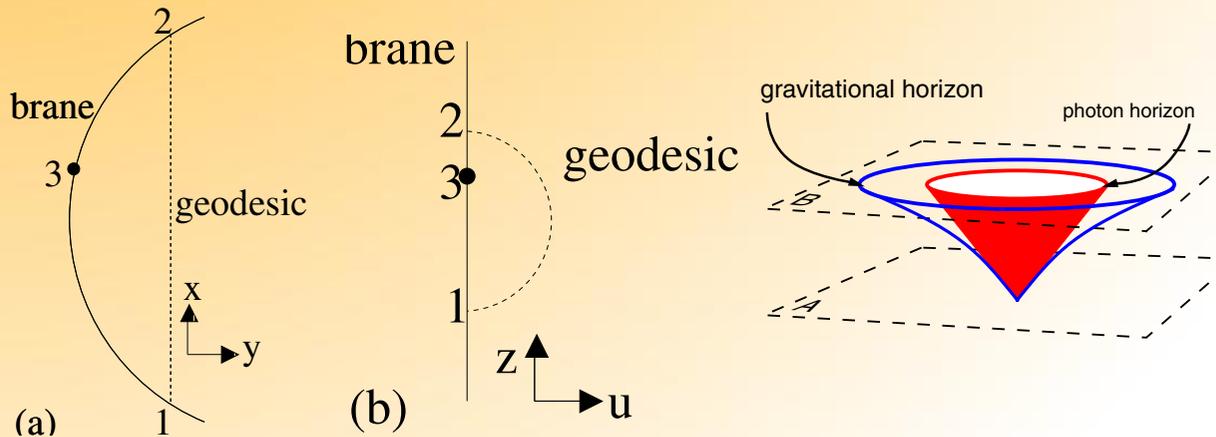
Standard cosmology: Universe homogenous over distances without causal contact (CMB)



- **Conventional solution:** Inflationary epoch in the early universe:

$$R(t) \propto \exp(\sqrt{\Lambda/3}t)$$

- **Alternative solution:** graviton shortcuts in the extra dimension D.J.H. Chung & K. Freese, 1999; G. Kaelbermann & H. Halevi, 1998; R.R. Caldwell & D. Langlois, 2001



What about
sterile neutrinos

?

SHOrtcut Bulk STERileS?

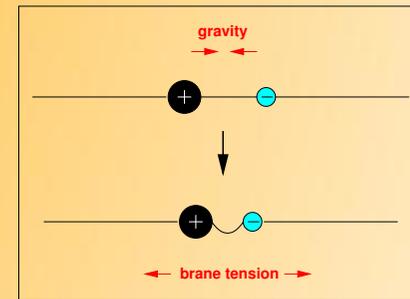


Microsoft: Impossible creatures

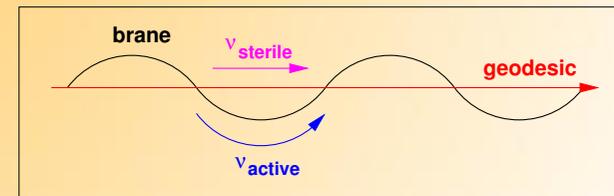
Bulk shortcuts of sterile neutrinos

3 mechanisms for microscopical brane buckles:

- Gravitational self attraction due to brane matter



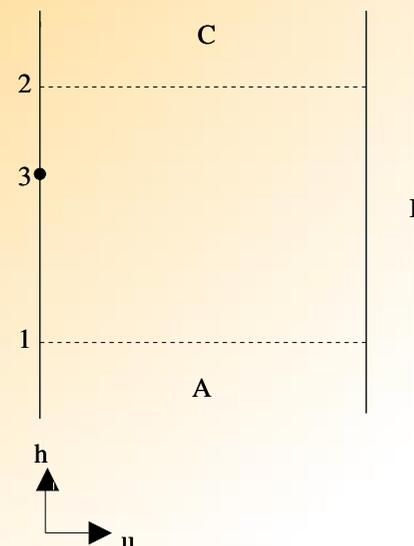
- Quantum fluctuations



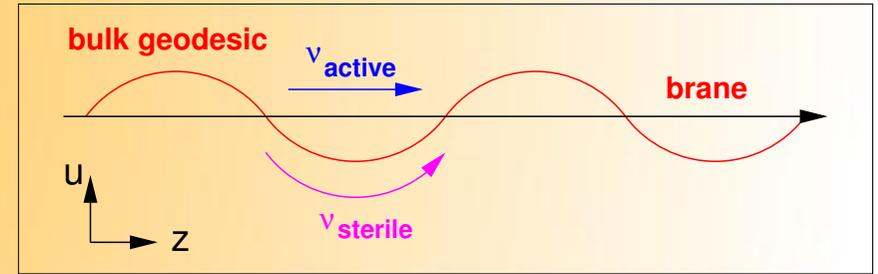
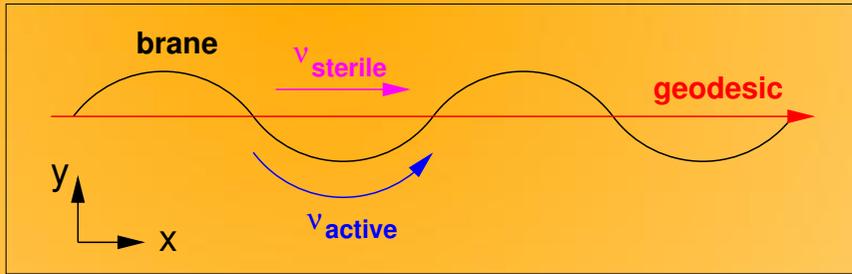
- Chung-Freese 2 brane system

$$ds^2 = dt^2 - [e^{-2ku} a^2(t) dh^2 + du^2]$$

D.J.H. Chung & K. Freese, 1999



A toy metric for bulk shortcuts



1+2 d Minkowski spacetime:

$$ds^2 = dt^2 - dx^2 - dy^2$$

Periodic brane: $y = \sin kx$

ν_s **bulk geodesic** (x, y -system):

$$y_{\text{bulk}} = 0, \quad x_{\text{bulk}} = t \Rightarrow$$

$$u_{\text{bulk}} = -\sin(kt),$$

$$z_{\text{bulk}} = \int_0^{t_f} \sqrt{1 + k^2 \cos^2 kt} dt$$

$$= \frac{\sqrt{(1+k^2)}}{k} \mathcal{E} \left(kt_f, \sqrt{\frac{k^2}{1+k^2}} \right)$$

$$z_{\text{bulk}} > z_{\text{brane}}$$

Ratio of travel times:

coordinate transformation $(x, y) \rightarrow (u, z)$

straight brane:

$$u = y - \sin kx \text{ and}$$

$$z = \int \sqrt{1 + k^2 \cos^2 kx} dx$$

\rightarrow (uz)-line element:

$$ds^2 = dt^2 - dz^2 - du^2 - \frac{2 \cos kx(z)}{\sqrt{1+k^2 \cos^2 kx(z)}} du dz$$

$\gamma/\nu_{\text{active}}$ **moving along the brane**

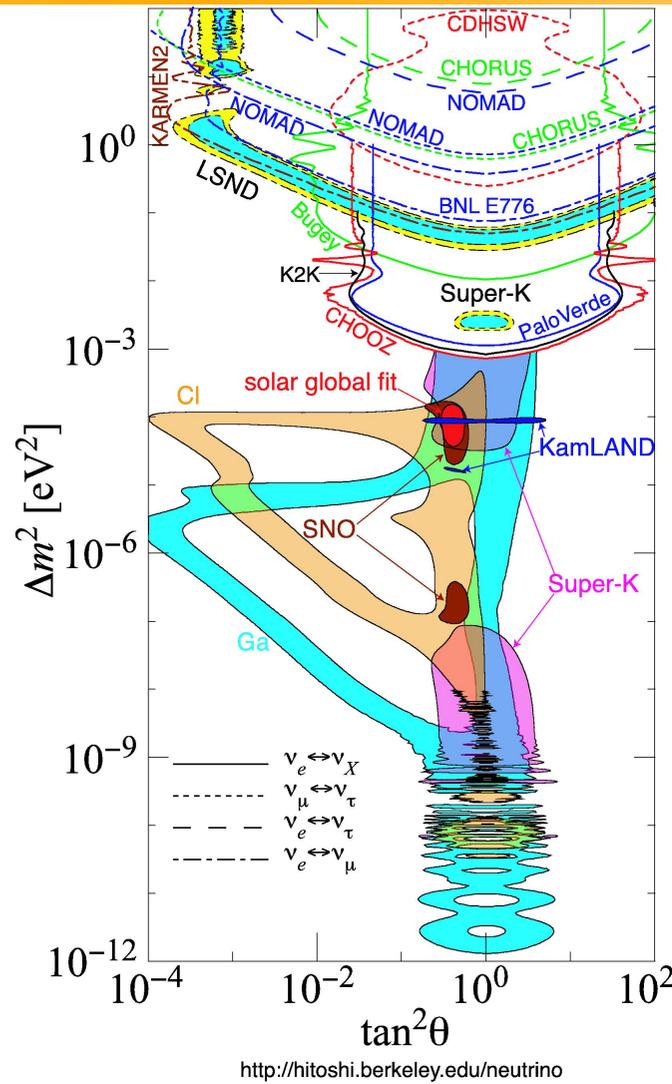
(u, z -system): $z_{\text{brane}} = t_f$

$$\epsilon = 1 - \frac{k t_f}{\sqrt{(1+k^2) \mathcal{E} \left(k t_f, \sqrt{\frac{k^2}{1+k^2}} \right)}}$$

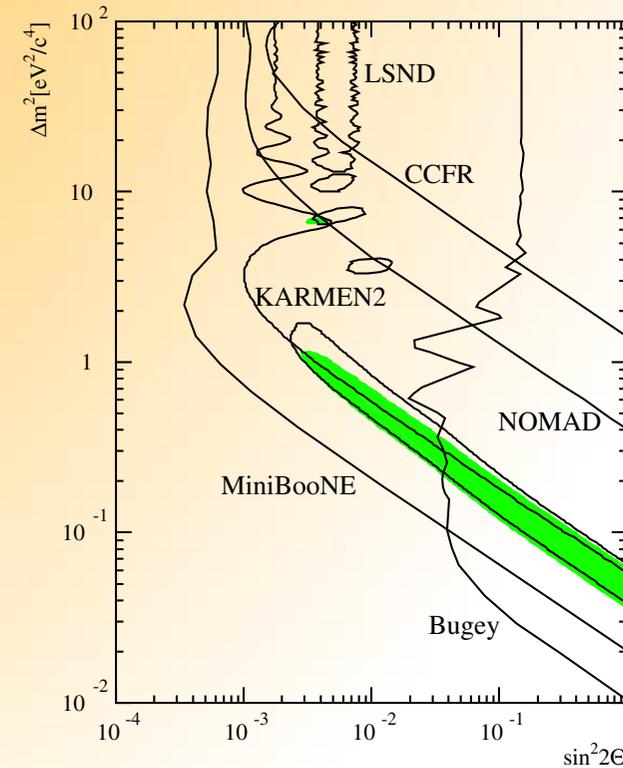
Toy metric allows for
apparent superluminal
propagation!

How
do bulk shortcuts
affect
neutrino oscillations ?

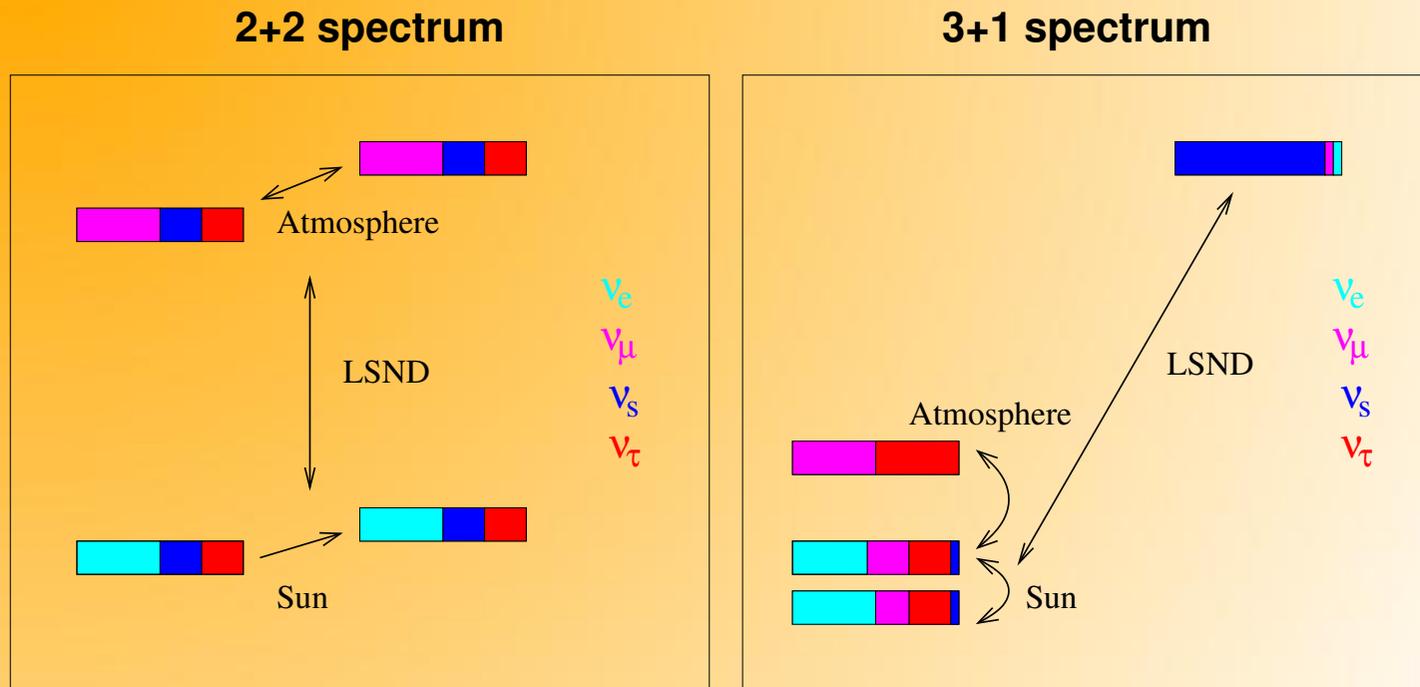
The LSND Dilemma



- 3 Δm^2 's \rightarrow 4 neutrinos!
- width of the Z-boson (LEP) \rightarrow 3 neutrinos!
- \rightarrow one **sterile neutrino?** (i.e. not coupling to the Z)



LSND and sterile Neutrinos



2+2 spectrum:

no oscillations of solar or atmospheric ν 's into steriles \rightarrow **excluded!**

3+1 spectrum:

BUGEY bound ($\nu_e \rightarrow \nu_{e\prime}$): $\sin^2 2\theta_{e\prime} = 4U_{e4}^2 (1 - U_{e4}^2)$

CDHS bound ($\nu_\mu \rightarrow \nu_{\mu\prime}$): $\sin^2 2\theta_{\mu\prime} = 4U_{\mu4}^2 (1 - U_{\mu4}^2)$

LSND ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$): $\sin^2 2\theta_{\text{LSND}} = 4U_{e4}^2 U_{\mu4}^2$

LSND is doubly suppressed! $\sin^2 2\theta_{\text{LSND}} \simeq \frac{1}{4} \sin^2 2\theta_{e\prime} \sin^2 2\theta_{\mu\prime} \rightarrow$ **excluded!**

Bulk shortcuts and neutrino oscillations

Evolution equation in flavor space:

$$i \frac{d}{dt} \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix} = H_F \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix}$$

Hamiltonian in the presence of bulk shortcuts:

$$H_F = + \frac{\delta m^2}{4E} \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix} + E \frac{\epsilon}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

⇒ **A Resonance exists at** $E_{\text{res}} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$

→ **choose** $E_{\text{res}} = 30 - 400 \text{ MeV} \leftrightarrow \epsilon \simeq 10^{-18} - 10^{-16}$

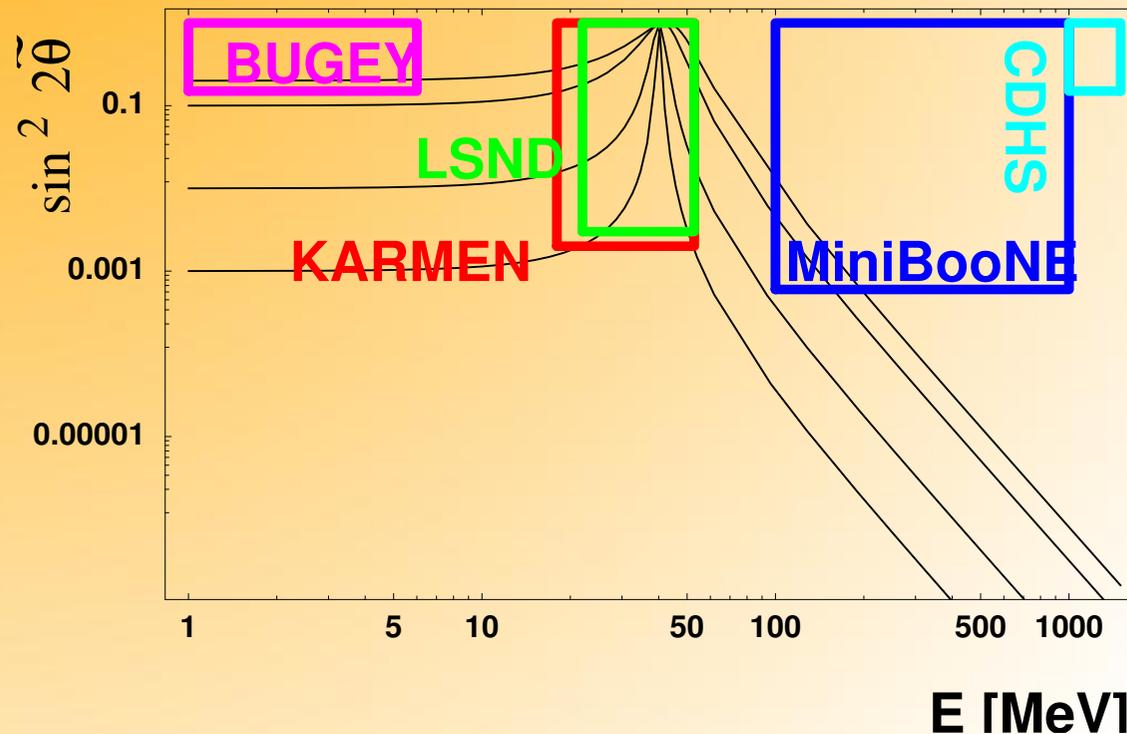
The active-sterile oscillation probability

$$P_{as} = \sin^2 2\tilde{\theta} \sin^2(\delta H D/2)$$

$$\sin^2 2\tilde{\theta} = \left[\frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \right]$$

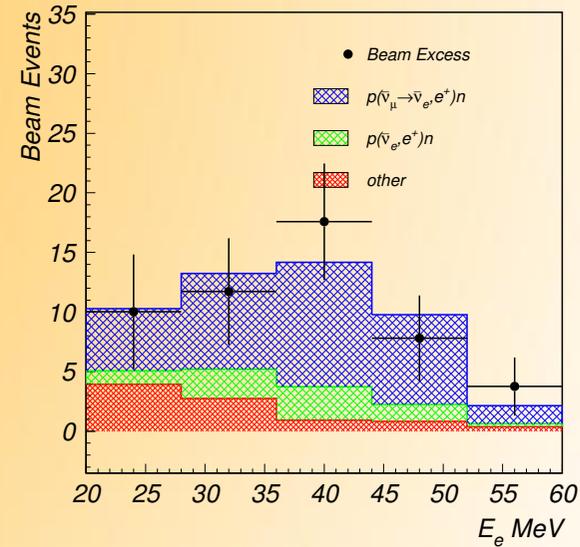
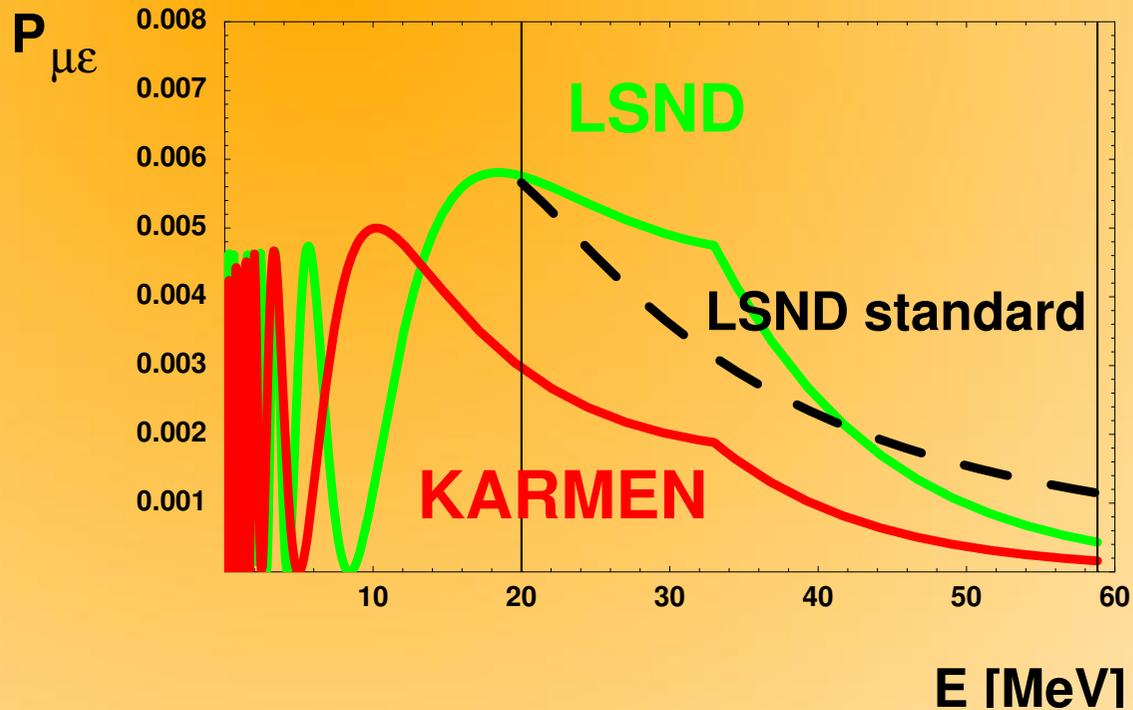
$$\delta H = \frac{\delta m^2}{2E} \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}$$

$$A = (E/E_{\text{res}})^2$$



Oscillations at $E \gg E_{\text{res}}$ (CDHS) are suppressed!

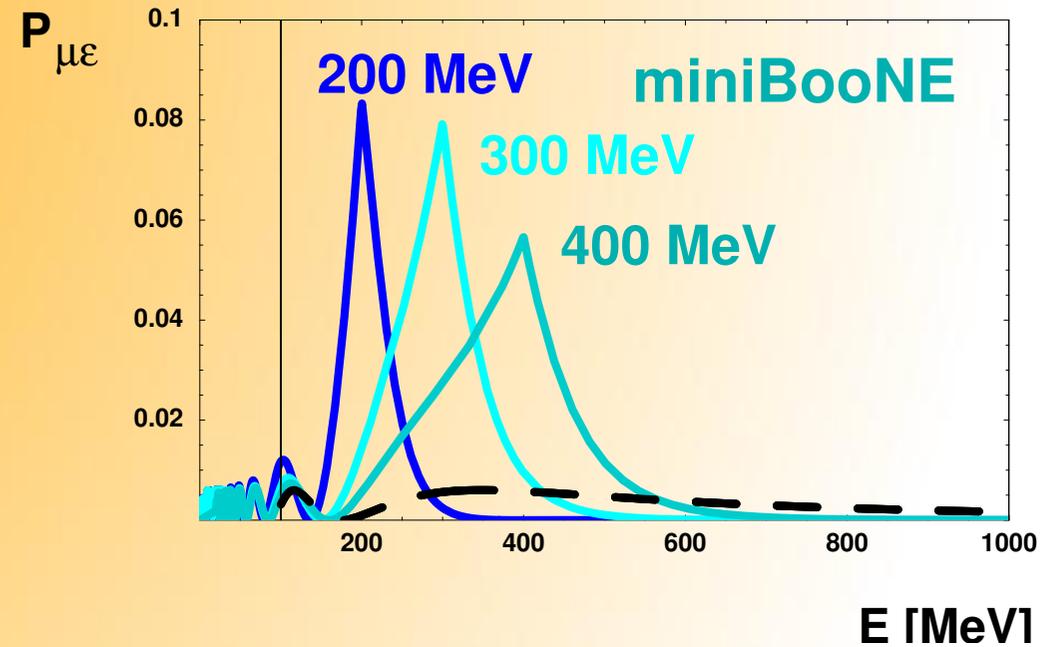
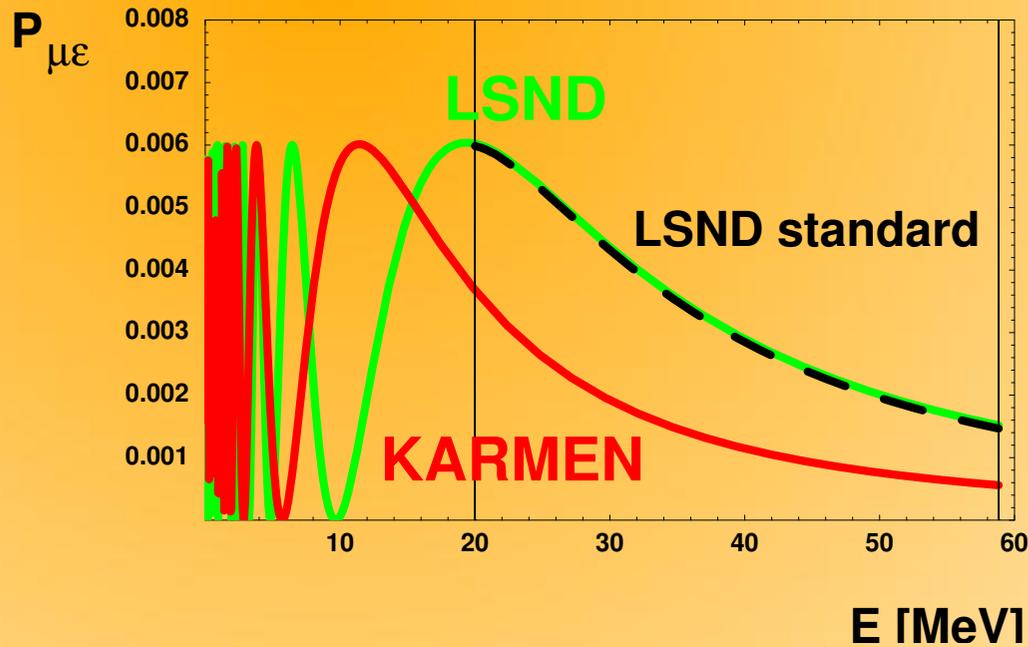
Scenario with low resonance energy



- $E_{\text{res}} = 33 \text{ MeV}$; $\sin^2 \theta_* = 0.01$;
 $\sin^2 2\theta = 0.9$; $\delta m^2 = 0.7 \text{ eV}^2$
- $P_{\text{LSND}} > P_{\text{KARMEN}}$

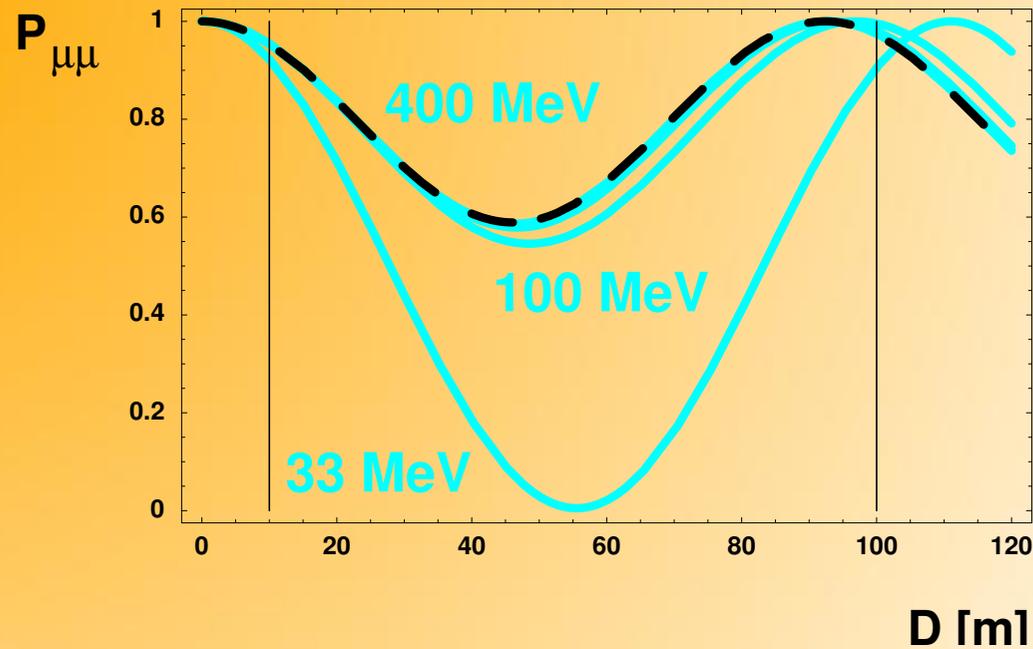
- good (better) fit of LSND spectrum
- no signal at miniBooNE!

Scenario with high resonance energy



- $E_{\text{res}} = 200 \text{ MeV}, 300 \text{ MeV}, 400 \text{ MeV}$; $\sin^2 \theta_* = 0.1$; $\sin^2 2\theta = 0.45$;
 $\delta m^2 = 0.8 \text{ eV}^2$
- good fit to LSND spectrum, $P_{\text{LSND}} > P_{\text{KARMEN}}$
- enhanced miniBooNE signal in the energy range 100-600 MeV

Consequences for a stopped pion source at SNS



- Low energy ν_{μ} disappearance experiment proposed at the Spallation Neutron Source at Oak Ridge or at Fermilab proton driver
- $E_{\text{res}} = 33 \text{ MeV}$; $\sin^2 \theta_* = 0.01$; $\sin^2 2\theta = 0.9$; $\delta m^2 = 0.7 \text{ eV}^2$
- $E_{\text{res}} = 100 \text{ MeV}, 200\text{-}400 \text{ MeV}$; $\sin^2 \theta_* = 0.1$; $\sin^2 2\theta = 0.45$; $\delta m^2 = 0.8 \text{ eV}^2$
- **strongly enhanced ν_{μ} depletion signal**

Big Bang Nucleosynthesis

Prediction of primordial abundances of light elements: major success of Big Bang Cosmology

Problem with sterile neutrinos:

ν oscillations populate extra species in early universe:

$$\rho_{\nu_s} = \frac{7}{8} \rho_\gamma$$

- \rightarrow faster expansion of the universe
- \rightarrow higher temperature for weak freezeout
- \rightarrow more neutrons \rightarrow larger ${}^4\text{He}$ abundance

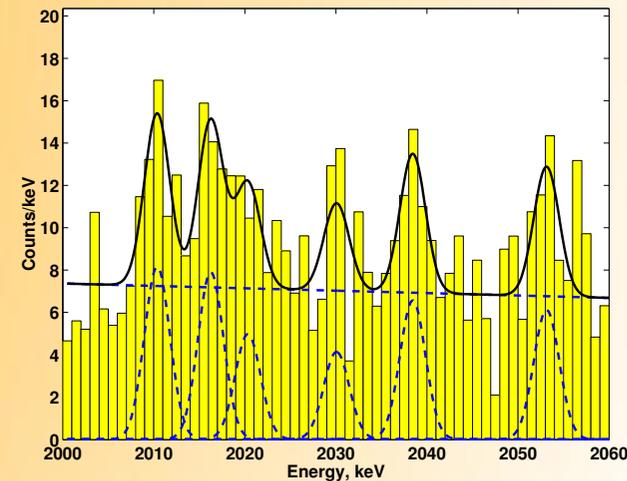
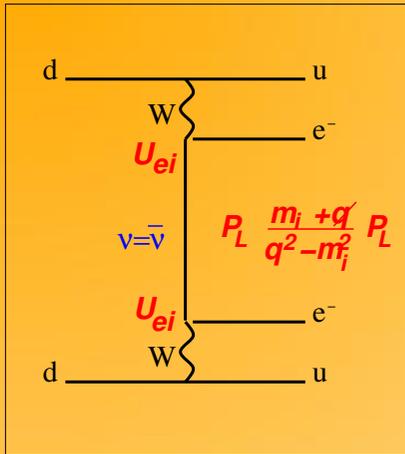
Bulk shortcut scenario:

- higher density: larger brane buckle effect due to gravitational attraction
- higher temperature: more quantum fluctuations
- higher density: more scattering off the brane in Chung-Freese scenario

All cases: larger $\epsilon \rightarrow$ smaller E_{res}

If $E_{\text{res}} \lesssim 3 \text{ MeV}$: oscillations suppressed

Neutrinoless double beta decay



$$\Gamma_{0\nu\beta\beta} \propto |m_{ee}|^2 = |\tilde{U}_{e4}^2 \tilde{m}_4 + \sum_{j=1,3} \tilde{U}_{ej}^2 m_j|^2$$

$m_{ee} = 0.1 - 0.9$ eV? (Klapdor-Kleingrothaus, Krivosheina, Dietz, Chkvorets 2004)

- $p_\nu \sim p_F \sim \mathcal{O}(100)$ MeV
- $p_\nu \sim E_{\text{res}} \Rightarrow \tilde{U}_{e4}$ may become large!
- large signals for $0\nu\beta\beta$ decay!

Tritium β decay/Cosmology ($E_\nu \simeq \mathcal{O}(\text{MeV})$): no enhancement/suppression due to smaller E_{res} – “could save Klapdor’s ass”

Supernova neutrinos

- Spectra: Bulk shortcut effects: the matter resonant conversion will be cut off above E_{res}
- Supernova cooling bounds can be avoided by choosing small A and large k without affecting $\epsilon \simeq (Ak/2)^2$
 - small compactification radius
 - KK states decouple
- R-process: $\nu_e \rightarrow \nu_s$ oscillations protect neutrons from being wiped out via $\nu_e n \rightarrow pe$
G.M. Fuller, 1999

LSND neutrino & keV warm dark matter

WDM solves:

- cuspy core problem of cold dark matter
[S.Dodelson, L. Widrow, 1993](#); [X.d. Shi, G.M. Fuller, 1998](#)
- induce observed velocities of radio pulsars
[G.M. Fuller, A. Kusenko, I. Mocioiu, S. Pascoli, 2003](#)

$$\delta m^2 \rightarrow \delta m^2 \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta} \ll \delta m^2?$$

However: small $\sin^2 2\theta$ seems not to fit LSND spectrum

Sterile neutrinos & the horizon problem

Are sterile neutrino shortcuts superior to graviton shortcuts?

- Bounds from precision experiments on gravitational square law do not apply
- Sterile neutrinos may couple more strongly (Homogeneity problem)

Causality Considerations

- Space-like brane path: $ds_{\text{brane}}^2 = dt^2 - dz^2 < 0$
can be time-like via the bulk: $ds_{\text{bulk}}^2 = dt^2 - dz^2 > 0$
- Brane geodesics are no bulk geodesics and vice versa!
small see, e.g. H. Ishihara, PRL 86 (2001) 381
- 3 possibilities to define Lorentz invariance
 - ds_{bulk}^2 invariant \rightarrow unlikely as extra dimensions are treated worse than lab rats (compactified + orbifolded/warped)
 - Neither ds_{bulk}^2 nor $ds_{\text{brane}}^2 \rightarrow$ Lorentz invariance violations on the brane, Bremsstrahlung?
 - ds_{brane}^2 invariant \rightarrow bulk shortcuts resemble of wormholes!
- ...possibility of time travel?

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 - ds_{bulk}^2 invariant \rightarrow **unlikely** as extra dimensions are treated worse than lab rats (compactified + orbifolded/warped)
 - **Neither** ds_{bulk}^2 **nor** $ds_{\text{brane}}^2 \rightarrow$ **Lorentz invariance violations** on the brane, Bremsstrahlung?
 - ds_{brane}^2 invariant \rightarrow **bulk shortcuts resemble of wormholes!**
- **...possibility of time travel?**
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 - Buy: Matt Visser - Lorentzian Wormholes, AIP press
 - Ignore insults like: *“Psycho-ceramics warning: Crackpots are politely requested to refrain from reading this paragraph”*

Wormholes and time machines

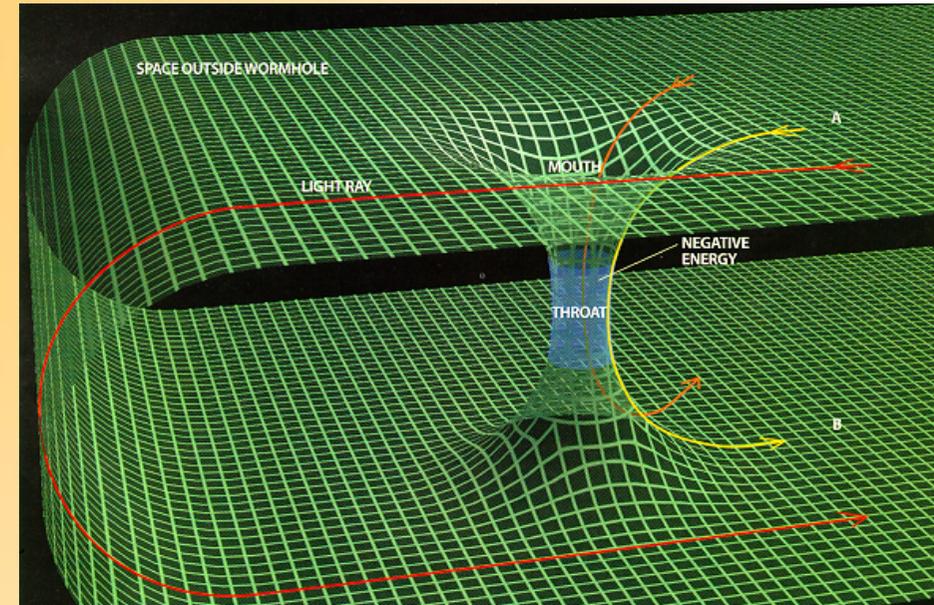
Matt Visser's 3 steps

1. Find a wormhole
(*"recipe for dragon stew"*)

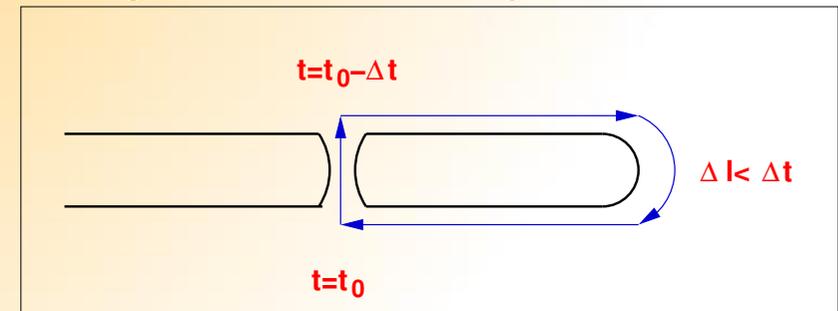


T. Roman, Scientific American

2. Introduce a time-shift



3. Bring the mouths together



Morris, Thorne, Yurtsever, PRL 61 (1988) 1446

Time machine proposals

- Thorne's Wormholes (1988)
- Gödel's rotating universe (1949)
- Van Stockums and Tiplers rotating cylinders (1937 & 1974)
- Gott's pair of moving cosmic strings (1991)
- Wheelers space time foam (1962)
- Kerr and Newman geometries (1973)
- Alcubierres warp drive (1994)

Obstacles:

- unphysically fast rotation → tipping Lorentz cones
- safely hidden behind event horizons
- exotic energy violating the energy conditions ($\sim \rho < 0$) → strong curvature
→ $T_{\mu\nu}$ diverges at “chronology horizon”

Timekeepers



“The Time Keepers were born at the end of time, entrusted with the safety of Time by He Who Remains. The Time Keepers were meant to watch over the space time continuum just outside of Limbo, and make sure the universe thrived.” [Marvel Comics](#)

“It seems that there is a chronology protection agency which prevents the appearance of closed time-like curves and so makes the universe safe for historians.”

[S.W. Hawking](#), The chronology protection conjecture, *Phys. Rev. D* 46 (1992) 603-611

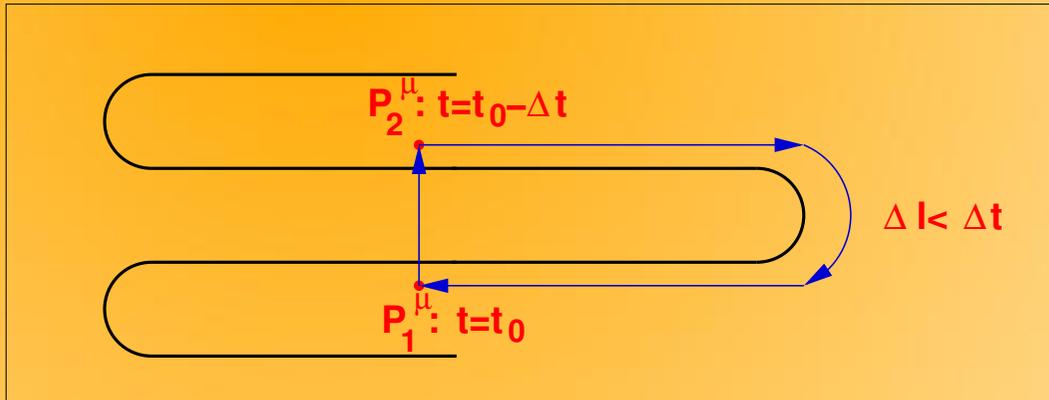
“Semi-classical wormholes and time machines are unstable”

[Roman Buniy](#), *Frontiers in Contemporary Physics III*



“Too pessimistic, If worms can do it, people can learn how!” [anonymous chatter](#)

Challenging the Timekeepers?



- Consider **two time-like world lines** on the brane: $P_1^\mu = (t, u = 0, z_0)$ and $P_2^\mu = P_1^\mu + \delta^\mu$ with the **space-like on-brane distance** $\delta^\mu = (0, 0, z_{\text{brane}} \equiv \Delta l)$
- Extreme case: $k \rightarrow \infty$
- \rightarrow **Bulk distance** of P_1^μ and $P_2^\mu \rightarrow 0$.
- Effective space-time: Minkowski metric of the brane at $u = 0$ with $P_1^\mu \equiv P_2^\mu$
- Add time-shift: \rightarrow Family of closed curves, parametrized by the parameter $\sigma \in [0, 1]$:

$$X^\mu(t, \sigma) = \sigma P_1^\mu + (1 - \sigma)(P_2^\mu)' = (t + (1 - \sigma)\Delta t, 0, z_{\text{brane}}),$$

- **Closed time-like curves arise as soon as $\Delta t > \Delta l \equiv z_{\text{brane}}$!**

Energy conditions

Does $T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & -\vec{p} \end{pmatrix}$ of $ds^2 = dt^2 - dz^2 - du^2 - \frac{2 A k \cos kx(z)}{\sqrt{1+A^2 k^2 \cos^2 kx(z)}} du dz$

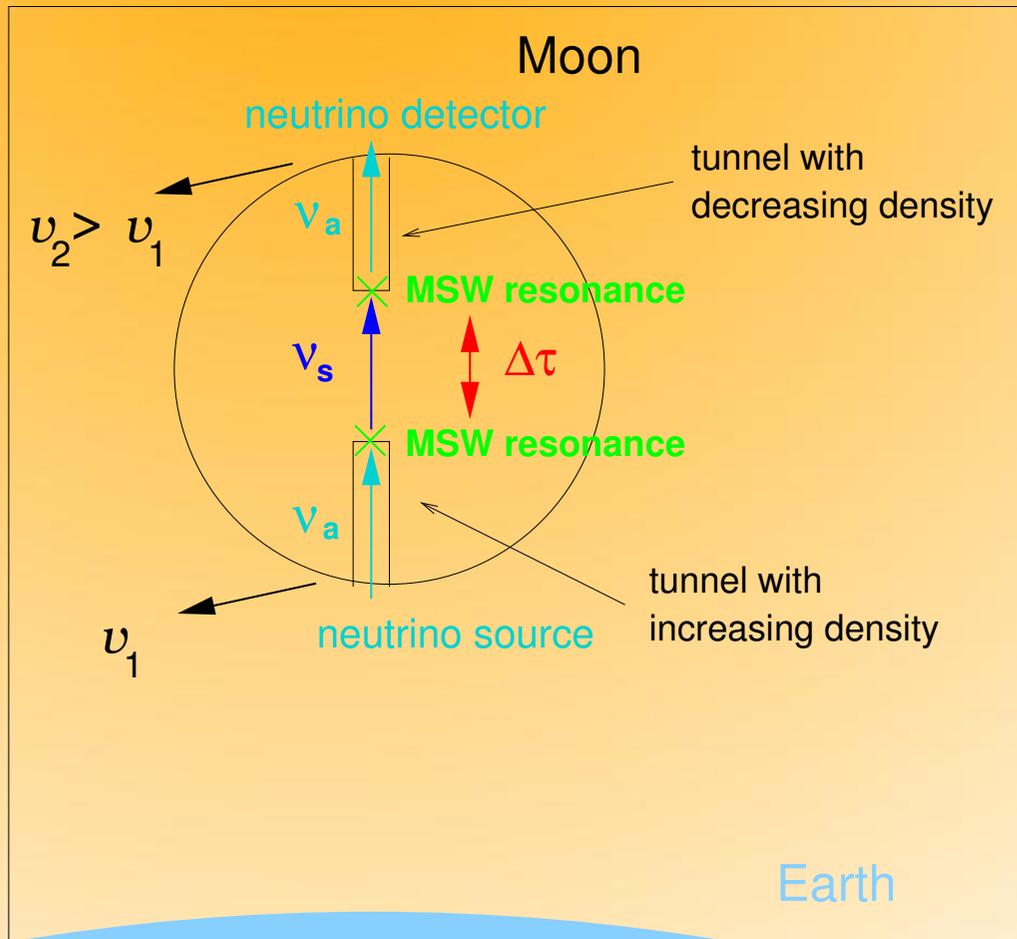
violate any of the following energy conditions?

NEC ($\rho + p_j \geq 0$), **WEC** ($\rho \geq 0$ & $\rho + p_j \geq 0$), **SEC** ($\rho + p_j \geq 0$ & $\rho + \sum_j p_j \geq 0$), **DEC** ($\rho \geq 0$ & $p_j \in [-\rho, +\rho]$)

- Of the **Christoffel symbols** $\Gamma_{\lambda\mu}^{\kappa} = \frac{g^{\kappa\nu}}{2} \left(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right)$
- **only** $\Gamma_{11}^1 = f(z) \frac{\partial f(z)}{z}$ & $\Gamma_{11}^2 = \frac{\partial f(z)}{z}$ **survive**
- \Rightarrow **In Ricci tensor** $R_{\mu\nu} = \frac{\partial \Gamma_{\mu\rho}^{\rho}}{\partial x^{\nu}} - \frac{\partial \Gamma_{\mu\nu}^{\rho}}{\partial x^{\rho}} + \Gamma_{\mu\rho}^{\sigma} \Gamma_{\sigma\nu}^{\rho} - \Gamma_{\mu\nu}^{\sigma} \Gamma_{\sigma\rho}^{\rho}$, only **terms** with $\mu\rho\sigma\nu = (1, 1, 1, 1)$ contribute which **cancel pairwise!**
- Thus $R = g^{\mu\nu} R_{\mu\nu} = 0$ and $T_{\mu\nu} = \frac{1}{8\pi G} (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) = 0$ **vanish identically!**
- **All energy conditions are trivially conserved!**

Neutrino time machine

- Use **twin paradox of rotational aligned earth-moon system!**
- Use **MSW matter effect to create sterile neutrinos!**



- $\Delta t = \Delta \left(\int_0^{t_{\text{Moon}}} \sqrt{1 - v^2} dt \right)$

- $\simeq t_{\text{Moon}} \cdot \frac{\Delta v^2}{2}$

- Integration over the earth-moon lifetime (4 billion years)

- **$\Delta\tau$ can be $\gtrsim O(\text{hours})!$**

Päs, Pakvasa, Weiler, in preparation

- **Experimental study of physics close to the chronology horizon**

Conclusions

- Bulk shortcuts may arise in extra dimensional theories
- Bulk shortcuts affect neutrino mixing and imply a new resonance
- Neutrino oscillations are suppressed for $E \gg E_{\text{res}}$
- LSND becomes compatible with BUGEY and CDHS ($E \gg E_{\text{res}}$)
- $E_{\text{res}} < 100$ MeV: no signal at miniBooNE but distorted LSND spectrum and enhanced oscillations at SNS
- $E_{\text{res}} \gg 100$ MeV: enhanced oscillations at miniBooNE
- BBN bound can be evaded
- Neutrinoless double beta decay may be enhanced
- Large signals possible at future ν_e (reactor) disappearance experiments
- R-process nucleosynthesis, supernova neutrinos, warm dark matter, horizon problem,...
- superluminal or even time travel